

## Small-scale inhomogeneity effects on coherent solar radio emission

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Submitted to ApJ

### ABSTRACT

Coherent radio emission mechanism of solar radio bursts is one of the most complicated and controversial topics in solar physics. To clarify the mechanism(s) of different types of solar radio bursts, (radio) wave excitation by energetic electrons in homogeneous plasmas has been widely studied via particle-in-cell (PIC) code numerical simulations. The solar corona is, however, inhomogeneous over almost all spatial scales. Inhomogeneities of the plasma could influence the emission properties of solar radio bursts. In this paper, we, hence, investigate effects of inhomogeneity (in the magnetic field, plasma density and temperature) of plasmas in the solar corona on radio wave emission by ring-beam distributed energetic electrons utilizing 2.5-dimensional PIC simulations. Both the beam and electron cyclotron maser (ECM) instabilities could be triggered with the presence of the energetic ring-beam electrons. The resultant spectrum of the excited electromagnetic waves presents a zebra-stripe pattern in the frequency space. The inhomogeneous density or temperature in plasmas influences the frequency bandwidth and location of these excited waves. Our results can, hence, help to diagnose the plasma properties at the emission sites of solar radio bursts. Applications of our results to the solar radio bursts with zebra-stripe pattern are discussed.

*Keywords:* Solar corona; Solar coronal radio emission; Radio bursts

### 1. INTRODUCTION

Radio emissions from the Sun are not only produced by incoherent processes of energetic electrons, but also via coherent processes involving kinetic instabilities of wave-particle, wave-wave interactions (Melrose 1991, 2017). Solar radio emission is, hence, the most suitable object to diagnose the physical state, acceleration and propagation processes of high-energy electrons in plasmas of solar activity. The solar radio emission mechanism, especially the coherent radio emission mechanism of solar radio bursts (Dulk 1985), is one of the most complicated and controversial topics in the solar physics. There has been a constant controversy between the two types of coherent emission mechanisms, i.e., "the plasma emission" (Ginzburg & Zhelezniakov 1958; Melrose 1970a,b; Zheleznyakov & Zaitsev 1970a,b) and "the electron cyclotron maser (ECM) emission" (Twiss 1958; Schneider 1959; Gaponov 1959), for the mechanism of solar radio bursts since a long time. For reviews, see, e.g., Aschwanden 2005; Melrose 2017.

Due to the kinetic nature of coherent radio emissions, fully kinetic Particle-in-Cell (PIC) numerical simulation has been widely applied to figure out mechanism of coherent radio emission from different space plasmas. For instance, Kasaba et al. 2001; Rhee et al. 2009; Umeda 2010; Thurgood & Tsiklauri 2015; Henri et al. 2019; Chen et al. 2022; Ni et al. 2020 investigated the excitation process of electromagnetic waves by the plasma emission mechanism in weakly magnetized plasmas with  $\omega_{ce} < \omega_{pe}$ , where  $\omega_{ce}$  and  $\omega_{pe}$  are the electron gyrofrequency and electron plasma frequency, respectively. The ECM emission properties

in strongly magnetized plasmas with  $\omega_{ce} > \omega_{pe}$  were explored by Pritchett 1984; Lee et al. 2011; Ning et al. 2021; Zhou et al. 2021; Yousefzadeh et al. 2022. Zhou et al. 2020, 2022 systematically compared the excitation efficiency of electromagnetic waves in differently magnetized plasmas via PIC simulations. While all PIC simulations in the above mentioned studies initially utilized homogeneous plasmas, where the magnetic field ( $\vec{B}$ ), density ( $n$ ) and temperature ( $T$ ) of particles are uniform throughout the whole simulation domain. There is, however, seldom a plasma immersed in a uniform magnetic field in realistic solar coronal environment. Additionally, plasmas, as we know, are always turbulent due to a diverse processes, including, e.g. reconnection Vlahos & Cargill 2009. Turbulence involves an energy cascade from large to small scales, and naturally creates inhomogeneities specially from large space scales to the smallest kinetic scales. As mentioned in the studies of Melrose 1975 and Winglee & Dulk 1986, the inhomogeneous nature of plasmas is quite important as it influences properties of the emission spectrum of radio bursts. Contribution of the inhomogeneity of background magnetic field as well as density and/or temperature in plasmas on the properties of the coherent radio emission, hence, needs to be further verified and expanded.

Pritchett & Winglee 1989 and Yao et al. 2021 investigated the effects of nonuniform magnetic field and inhomogeneous background plasma density on the ECM and plasma emission processes, respectively, via PIC simulations. However not only the background plasma was ignored but also the background magnetic field ( $\vec{B} = [0, B_y, 0]$ ) did not satisfy  $\nabla \cdot \vec{B} = 0$  in the study of Pritchett & Winglee 1989. Yao et al. 2021, however, considered inhomogeneity of the background plasma density in the direction along the background magnetic field only with a localized homogeneous energetic electron beam. The background magnetic field, background plasma temperature and density, temperature of the (localized) energetic electron beam are uniform in that study. The setups applied by Pritchett & Winglee 1989 and Yao et al. 2021 would lead to deviations from the equilibrium state of plasmas. There should be  $\vec{J} \times \vec{B} = \nabla P$  in a magnetohydrodynamics (MHD) equilibrium plasma, where  $\vec{J}$  and  $P$  are the current density and pressure in plasma, respectively. Disequilibrium might trigger other instabilities in plasmas, which would mislead us about the ECM and plasma emission processes. There are, however, very few analytical kinetic equilibria of plasmas because of the difficulty of solving exactly the nonlinear integro-differential system of the Vlasov-Maxwell equations. In this study we, hence, consider a MHD equilibrium initial setup for further investigation of wave excitation.

To our knowledge, there is still no fine observation to provide a precise distribution of energetic electron in the solar coronal. Theoretical analysis and numerical simulation have, however, proved that ring-beam momentum distribution can be formed in the presence of a quasi-perpendicular shock or magnetic reconnection Vlahos & Sprangle 1987; Vlahos 1987; Bessho et al. 2014; Shuster et al. 2014, 2015. Additionally, ring-beam distributed energetic electrons have been applied in our previous study for wave excitation in homogeneous plasmas (Zhou et al. 2020). Applying ring-beam distribution in this study will benefit to comparison of wave excitation properties between homogeneous and inhomogeneous plasmas. Energetic electrons are still considered to follow a ring-beam distribution in this study.

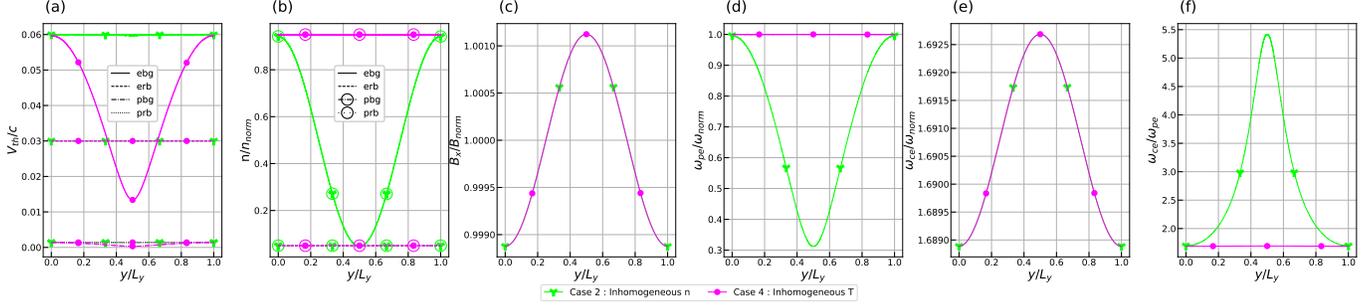
In this paper, we, hence, investigated the wave excitation by ring-beam distributed energetic electrons in inhomogeneous magnetized equilibrium plasmas, where the disequilibrium introduced by the inhomogeneous magnetic field is balanced by either inhomogeneous density or inhomogeneous temperature of the background plasma. 2.5-dimensional PIC simulations were utilized for the purpose of this study. Setup of this inhomogeneous magnetized equilibrium plasmas is shown in Section 2. The main results of these simulations are presented in Section 3. We draw our conclusions and discuss applications of our results to solar radio bursts with zebra-stripe pattern in Section 4.

## 2. SETUP OF PIC SIMULATIONS

We perform this study with the fully kinetic PIC code ACRONYM (Kilian et al. 2017), a fully relativistic electromagnetic code tuned for the study of kinetic-scale plasma wave phenomena and interactions in collisionless plasmas (see, e.g., Ganse et al. 2012; Schreiner et al. 2017; Muñoz & Büchner 2018; Zhou et al. 2020; Yao et al. 2021). We use the code in two spatial dimensions and three dimensions in velocity, electromagnetic fields (i.e., 2.5-dimensional or 2.5D). All quantities in simulations are solved in Cartesian-coordinate spatial-temporal space with Gaussian CGS units, and all simulations have the same spatial and time resolution. In particular, the grid cell size is  $\Delta x = \Delta y = D_e$  (where  $D_e$  is the Debye length of electrons at the boundaries of the y-axis) with  $N_x \times N_y = 1024 \times 4096$  grid points in the  $x - y$  plane of the 2D simulation box. Periodic boundary condition is applied to the boundaries of each axis. The timestep ( $\Delta t$ ) in our simulations is determined by the inherent length and timescale requirements in a fully kinetic PIC code, i.e., the Courant-Friedrichs-Lewy (CFL) condition for the speed of light  $c$ .

For our first investigation of the plasma inhomogeneity effects on the wave excitation, a simple inhomogeneous magnetic field following  $\vec{B} = [B_x(y), 0, 0]$ , with a magnetic field gradient along its perpendicular direction, i.e., the y-axis, is applied in this study. And

$$B_x(y) = B_0 \left[ \left( 1 + \frac{B_e}{B_0} \right) + (\eta - 1) \cos^2 \left( \frac{y}{L_y} \pi \right) \right] \quad (1)$$



**Figure 1.** Initial  $y$ -axis distribution of different (normalized) parameters: panel (a) and (b) — thermal velocity and number density of different particle species. The solid, dashed, dashdot, and dotted lines are separately for background electrons (*ebg*), energetic ring-beam electrons (*erb*), *pbg*-background protons (*pbg*), and *prb*-background protons (*prb*), respectively. Panel (c) — magnetic field  $B_x$ , panel (d) — electron plasma frequency  $\omega_{pe}$ , panel (e) — electron gyrofrequency  $\omega_{ce}$ , panel (f) — the ratio of the electron gyrofrequency and plasma frequency  $\omega_{ce}/\omega_{pe}$ . In each panel, different colors are used to distinguish different cases.

where  $L_y = N_y \Delta y$  is the  $y$ -axis size of the simulation domain. Value of  $\eta$  determines the  $y$ -axis location of the extremum of  $B_x(y)$  as well as the total magnetic field strength  $|B|$ . The maximum (minimum)  $|B|$  is located at the center (boundaries) of the  $y$ -axis with  $\eta < 1$  ( $\eta > 1$ ), while  $\eta = 1$  indicates a uniform magnetic field throughout the simulation domain. We consider  $\eta = 0.1$  as well as  $B_0 = 1.2 \text{ G}$ ,  $B_e = 400B_0$  in this study. The maximum  $|B|$  is about  $\sim 480 \text{ G}$ , which is the typical magnetic field strength of active regions on the Sun (Aschwanden 2005). Value of  $|B|$  along the  $y$ -axis is shown in panel (c) of Fig. 1. And consequently the maxima of the derivatives in Eq. (1) is  $\sim 10^{-3} \text{ G/cm}$ , which is over four orders of magnitude larger than that of the magnetic field model for active regions in the Sun, see Eq.(1.4.2) in Aschwanden 2005, although strength variance of this magnetic field is only  $\sim 1 \text{ G}$ . Taking account of turbulence and the small space scale covered by PIC simulations, as we mentioned in Sect.1, magnetic field with such "huge" derivative might exist at the small kinetic scales.

For a (MHD) equilibrium plasma environment magnetized by a inhomogeneous magnetic field, there should be

$$\begin{aligned}
 \nabla \cdot \vec{B} &= 0 \\
 \vec{J} \times \vec{B} &= \nabla P_{th} \\
 \nabla \times \vec{B} &= \frac{4\pi}{c} \vec{J}
 \end{aligned} \tag{2}$$

where  $\vec{J}$  and  $P_{th} = \sum_j n_j k T_j$  are the current density and thermal pressure in plasma, respectively.  $k$  is the Boltzmann constant.  $n_j$  and  $T_j$  are the number density and temperature of different particle species, respectively. And  $j$  is used to distinguish different particle species.

Four species of particles are employed in each simulation, i.e., energetic electrons (*erb*), background thermal electrons (*ebg*), *prb*-background protons (*prb*), and *pbg*-background protons (*pbg*), see panel (a) and (b) in Fig.1.  $j$  in the above equations can, hence, be *ebg*, or *erb*, or *pbg*, or *prb*. And there are, hence,  $n_p = n_{pbg} + n_{prb} = n_{ebg} + n_{erb}$ , where  $n_p$ ,  $n_{ebg} = n_{pbg}$  and  $n_{erb} = n_{prb}$  are the number density of background protons, background electrons and energetic electrons, respectively, to maintain the global charge neutrality. For physically realistic results, the proton-to-electron mass ratio has been chosen as  $m_p/m_e = 1836$ . And we consider temperature-isotropic plasmas. Density and temperature of the energetic electrons and *prb*-background protons are initially homogeneous throughout the whole simulation domain in each simulation. For an equilibrium plasma environment, where has  $P_B + P_{th} = \text{constant}$ , variation of  $P_B$  (magnetic pressure) has to be balanced by thermal pressure  $P_{th}$  via either inhomogeneous density (Case 2) or inhomogeneous temperature (Case 4) of both background electrons and *pbg*-background protons. In other words, density and temperature of the background electrons and *pbg*-background protons are kept the same in each grid cell of simulation. For instance, if the density of the background electrons and *pbg*-background protons are nonuniform, their temperatures will be homogeneous (Case 2). If inhomogeneous temperature is applied to the background electrons and *pbg*-background protons, they will be homogeneously distributed initially over the whole simulation domain (Case 4).

Momentum ( $\vec{u} = \gamma\vec{v}$ , where  $\gamma = 1/\sqrt{1 - v^2/c^2} = \sqrt{1 + u^2/c^2}$ ) of the energetic electrons initially follow a ring-beam distribution (Umeda et al. 2007; Lee et al. 2011; Kainer & MacDowall 1996; Zhou et al. 2020), i.e.,

$$\begin{aligned} F_{rb}(u_{\parallel}, u_{\perp}) &= F_{rb\parallel}(u_{\parallel})F_{rb\perp}(u_{\perp}) \\ F_{rb\parallel}(u_{\parallel}) &= \frac{1}{\sqrt{2\pi}u_{th\parallel}} \exp\left[-\frac{(u_{\parallel} - u_{d,erb,\parallel})^2}{2u_{th\parallel}^2}\right] \\ F_{rb\perp}(u_{\perp}) &= \frac{1}{2\pi u_{th\perp}^2 A_{\perp}} \exp\left[-\frac{(u_{\perp} - u_{d,erb,\perp})^2}{2u_{th\perp}^2}\right] \end{aligned} \quad (3)$$

where  $u_{\parallel}, u_{\perp}$  are the particle momenta along and perpendicular to the background magnetic field  $\vec{B}$ , i.e., the parallel and perpendicular directions are along the x-axis and y-axis, respectively. Initially we set  $u_{th\parallel} = u_{th\perp} = 0.03c$  and  $u_{d,erb,\parallel} = u_{d,erb,\perp} = 0.47c$ , correspondingly the average initial kinetic energy of energetic ring-beam electrons is  $100keV$  with a Lorentz factor  $\gamma_{d,erb} = 1.2$ .  $100keV$  is often considered for energetic electrons in solar activity. While  $A_{\perp}$  in Eq. (3) is the normalization constant

$$A_{\perp} = \exp\left[-\frac{u_{rb\perp}^2}{2u_{th\perp}^2}\right] + \sqrt{\frac{\pi}{2}} \frac{u_{rb\perp}}{u_{th\perp}} \operatorname{erfc}\left[-\frac{u_{rb\perp}}{\sqrt{2}u_{th\perp}}\right] \quad (4)$$

Initial momentum distribution of *prb*-background protons is non-drifting Maxwellian with a uniform thermal velocity  $u_{th,erb} = 0.0014c$  in each simulation. Initial momentum distribution of background electrons and *pbg*-background protons are, however, drifting Maxwellian, i.e.,

$$f_{ebg/pbg}(u_x, u_y, u_z) = C \exp\left[-\frac{u_{x,ebg/pbg}^2 + u_{y,ebg/pbg}^2 + (u_{z,ebg/pbg} - u_{d,z,ebg/pbg})^2}{2u_{th,ebg/pbg}^2}\right] \quad (5)$$

where  $C$  is the normalization constants.  $u_{th,ebg} = 0.06c$  and  $u_{th,pbg} = 0.0014c$  are the maximal thermal velocity of the background electrons and protons, respectively, see panel (a) in Fig. 1, which indicates that temperature of the background electrons  $T_{ebg}$  and protons  $T_{pbg}$  are the same. While  $u_{d,z,ebg}$  and  $u_{d,z,pbg}$  are the drift velocity (in the out-of-plane direction along z-axis) of the background electrons and protons. The presence of  $u_{d,z,ebg}$  and  $u_{d,z,pbg}$  are due to the gradient drift of particles (Zhou et al. 2015) in inhomogeneous magnetic field Eq. (1). There are, hence,  $u_{d,z,ebg}/T_{ebg} = -u_{d,z,pbg}/T_{pbg}$  as well as  $\vec{J} = n_{ebg}q_e u_{d,z,ebg} + n_{pbg}q_p u_{d,z,pbg}$ , where  $q_e$  and  $q_p$  are the charge of electrons and protons, respectively. The maximum value of  $u_{d,z,ebg}$  is  $1.1 \times 10^{-4} c$  and  $2.5 \times 10^{-5} c$  in Case 2 and Case 4, respectively. Current due to the drifting motion of the energetic ring-beam electrons along the background magnetic field is numerically compensated initially to eliminate effects of large net current on wave excitation (Melrose 1986; Matsumoto & Omura 1993; Henri et al. 2019).

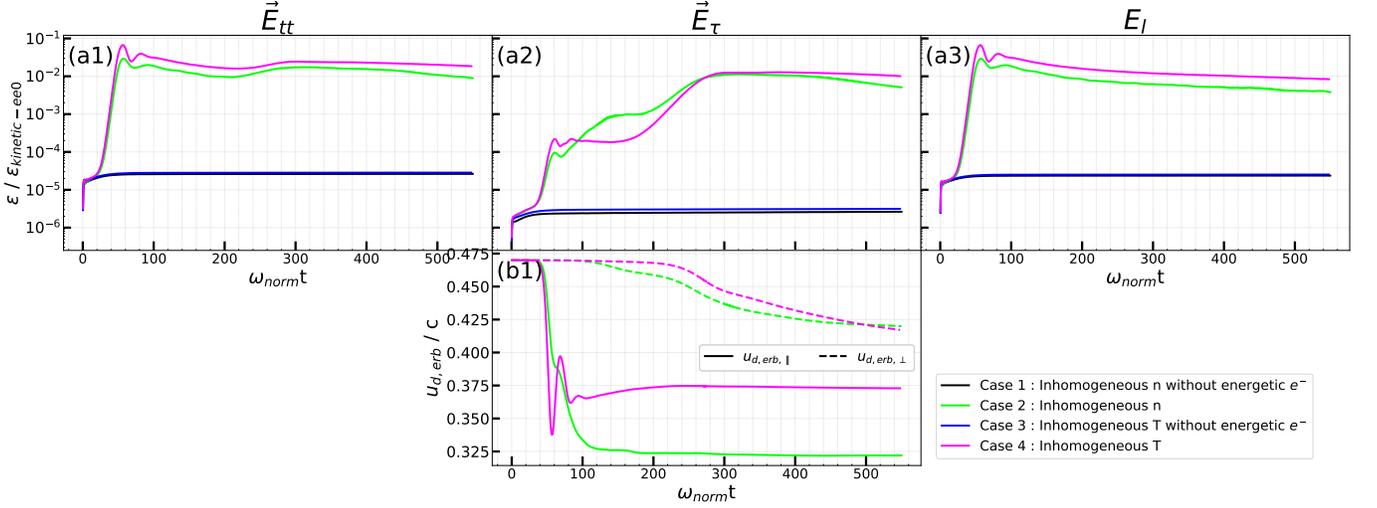
To increase the particle number per cell for the energetic ring-beam electrons, we set the macrofactor of the energetic ring-beam electrons being one quarter of that of background electrons. At the y-axis boundary of the simulation domain, for example, the number of macroparticles per cell is 950 for the background electrons and 200 for the energetic ring-beam electrons, where  $n_{erb}/(n_{erb} + n_{ebg})$  is, hence, 0.05, see panel (b) in Fig. 1. This density ratio was often applied in previous related investigations (see, e.g., Lee et al. 2011; Zhou et al. 2020).

Normalization of different parameters are as follows:  $\omega_{norm}$  is the normalization of frequency, which is equal to the maximal  $\omega_{pe}$  located at the boundaries of the y-axis in both plasmas (see panel d of Fig. 1). Correspondingly normalization of the particle number density is  $n_{norm} = \omega_{norm}^2 m_e / (4\pi e^2)$ , where  $e$  is the charge of electrons. Time, velocity and distance are normalized by  $1/\omega_{norm}$ ,  $c$  and  $c/\omega_{norm}$ , respectively.  $B_{norm}$ , being equal to the mean value of  $B_x(y)$  in Eq. (1), is the normalization of the electric and magnetic field strength. All kinds of energy are normalized by the initial kinetic energy of energetic ring-beam electrons  $\mathcal{E}_{kinetic-ee0}$ .

### 3. RESULTS

#### 3.1. Energy Evolution

Fig. 2 shows energy evolution of the total  $\vec{E}_{it} = (E_x, E_y, E_z)$  (panel a1), the transverse  $\vec{E}_{\tau} = \vec{E} \times \vec{k}/|\vec{k}|$  (panel a2), as well as the longitudinal  $E_l = \vec{E} \cdot \vec{k}/|\vec{k}|$  (panel a3) electric fields of waves in the simulation domain. Note that we refer to the properties of the transverse and longitudinal electric fields as those of the electromagnetic and electrostatic waves, respectively. Furthermore we also did (test) simulations for thermal plasmas with the same parameters as the above mentioned plasmas but without energetic electrons (i.e., pure thermal plasmas, Case 1 and Case 3). With top panels of Fig. 2, one can see that there is almost no electric



**Figure 2.** Energy evolution of the total ( $\vec{E}_n$  in panels a1), transverse ( $\vec{E}_\tau$  in panel a2) and the longitudinal ( $E_l$  in panel a3) electric fields of waves in the whole simulation domain. Panel (b1) presents evolutions of the bulk (or average) drift momenta in the directions along ( $u_{d,erb,||}$ , solid line) and perpendicular ( $u_{d,erb,\perp}$ , dashed line) to the ambient magnetic field  $\vec{B}$  for the energetic ring-beam electrons, Different colors in each panel are used to distinguish different cases.

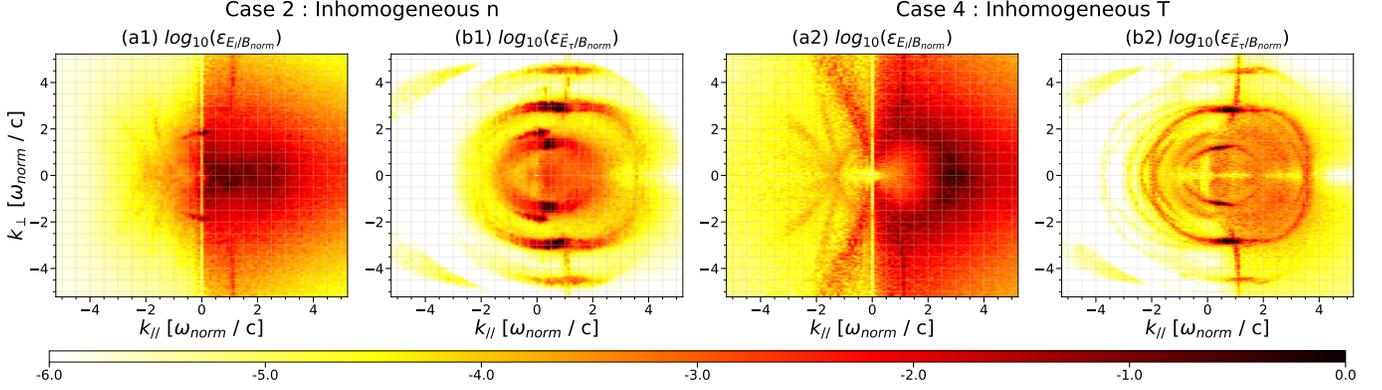
energy gain in the thermal plasmas, while the electric energy in plasmas with energetic electrons are more than three orders of magnitude larger than that in its corresponding thermal plasmas. These results indicate that the initial setups in this study stay in equilibria as well as there are indeed wave excitations instead of numerical noise in plasmas with energetic ring-beam electrons.

Panel (b1) of Fig. 2 presents evolutions of the bulk drift momenta in the directions along ( $u_{d,erb,||}$ ) and perpendicular ( $u_{d,erb,\perp}$ ) to the background magnetic field  $\vec{B}$  for the energetic ring-beam electrons. As we know, the beam and ECM instabilities (leading to the plasma emission and the ECM emission, respectively) are, in general, triggered by the free energy of energetic ring-beam electrons in the directions along (i.e.,  $u_{||}\partial F_{rb}(u_{||}, u_{\perp})/\partial u_{||} > 0$ ) and perpendicular (i.e.,  $\partial F_{rb}(u_{||}, u_{\perp})/\partial u_{\perp} > 0$ ) to the ambient magnetic field  $\vec{B}$ , respectively (Melrose 2017). Decrease of  $u_{d,erb,||}$  and  $u_{d,erb,\perp}$  indicates the free energy release in the corresponding direction. Evolution of  $u_{d,erb,||}$  and  $u_{d,erb,\perp}$  can, hence, give us insights on ideas, e.g., how fast these two instabilities reach their saturations, which instability is more efficient to release its free energy and trigger wave excitation, etc.

For each considered case, comparing panel (a1), (a2) and (a3) of Fig. 2, one can find that energy gain of the electric field is mainly dominated by the electrostatic waves, i.e.,  $E_l$  before  $\omega_{norm}t = 200$ . While after  $\omega_{norm}t = 200$ , electric energy of the electromagnetic waves ( $\vec{E}_\tau$ ) takes another increase and reaches the same energy level as the electrostatic waves. Combining the evolution of the parallel and perpendicular bulk momenta for the energetic ring-beam electrons (panel b1 of Fig. 2) with the electric energy evolution of the electrostatic and electromagnetic waves (panel a3 and a2 of Fig. 2, respectively), one can also discover that the first electric energy enhancement of the electromagnetic waves strongly correlates with the excitation of the electrostatic waves ( $E_l$ ) as well as the decrease of the  $u_{d,erb,||}$  before  $\omega_{norm}t = 100$ . While the second electric energy increase of the electromagnetic waves occurs together with the decrease of the  $u_{d,erb,\perp}$  after  $\omega_{norm}t = 200$ . These correspondences indicate the first (second) excitation of the electromagnetic waves should be attributed to the beam (ECM) instability. Furthermore, the second saturation (or peak) of the electromagnetic waves is almost two orders of magnitude larger than its first one, which manifests that the ECM instability can more efficiently excite the electromagnetic waves than the beam instability in plasmas, same as the previous studies, e.g., Zhou et al. 2022. Among different cases, energy of both the electrostatic and the electromagnetic waves are slightly stronger in plasma with inhomogeneous temperature (Case 4) than those in the inhomogeneous-density plasma (Case 2) during most of the simulation time but beside of the time period  $100 < \omega_{norm}t < 260$ . Among  $100 < \omega_{norm}t < 260$ , the stronger energy of the electromagnetic waves presenting in the inhomogeneous-density plasma (Case 2) is due to its earlier excitation onset of the electromagnetic waves by the ECM instability, indicated by the decrease of the  $u_{d,erb,\perp}$ .

### 3.2. Properties of excited waves

Fig. 3 presents the electric energy distribution in the  $k_{||} - k_{\perp}$  space of the electrostatic ( $E_l$ , panels a1 and a2) and electromagnetic ( $\vec{E}_\tau$ , panels b1 and b2) waves in plasmas with inhomogeneous density (Case 2, panels a1 and b1) or inhomogeneous temperature (Case 4, panels a2 and b2). Due to the symmetry in the direction perpendicular to the ambient magnetic field  $\vec{B}$ ,  $k_{||} - k_{\perp}$  spectrum



**Figure 3.**  $k_{\parallel} - k_{\perp}$  spectrum for the electric field of the electrostatic ( $E_i$ , panels a1 and a2) and electromagnetic  $\vec{E}_{\tau}$ , panels b1 and b2) waves over the whole simulation period ( $\omega_{\text{norm}}t = 0 \sim 546$ ) in plasmas with inhomogeneous density (Case 2, panels a1 and b1) or inhomogeneous temperature (Case 4, panels a2 and b2).

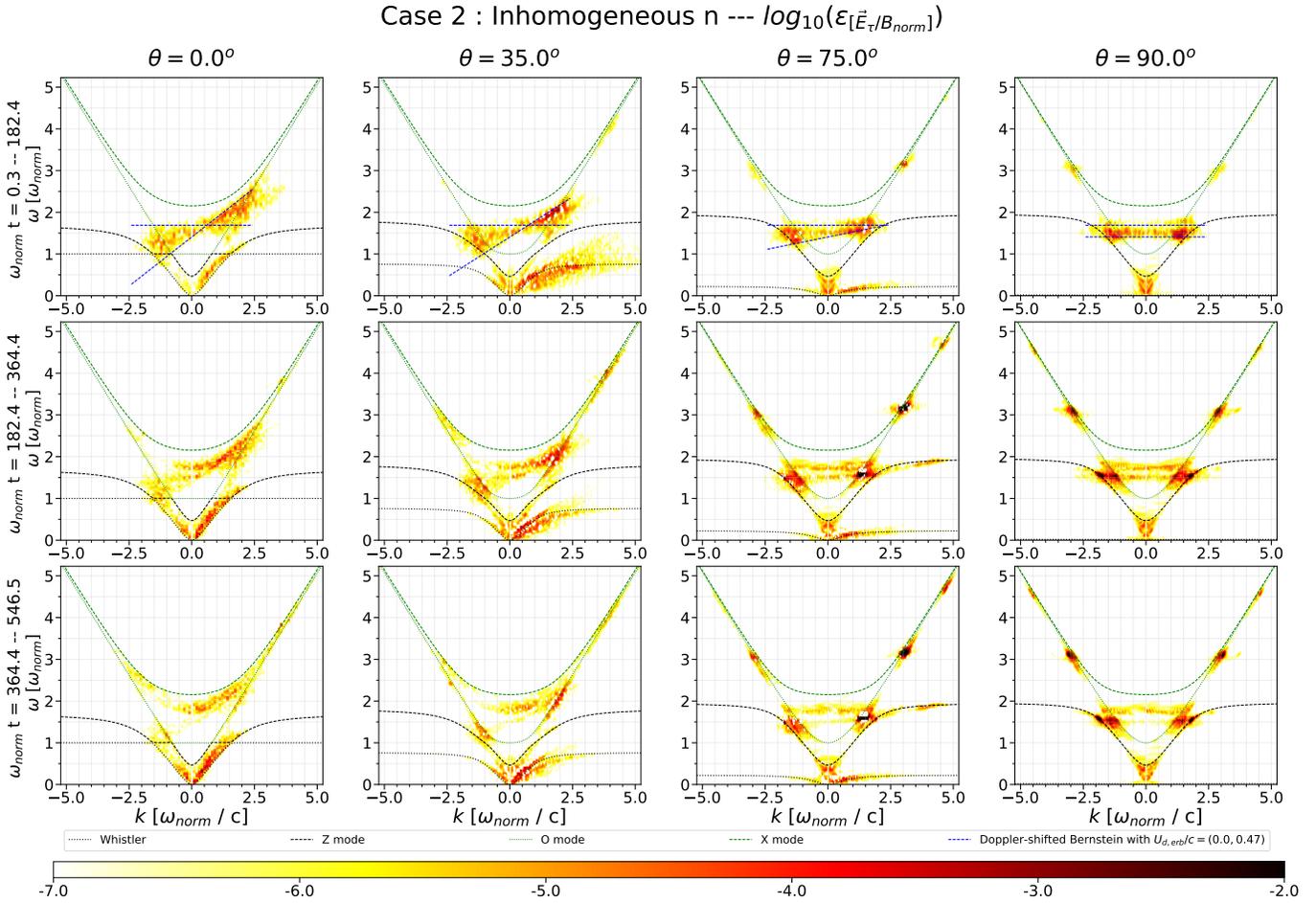
of both the electrostatic and electromagnetic waves exhibit a symmetry in the  $k_{\perp}$  direction. We would, hence, concentrate on the waves with  $k_{\perp} > 0$  in the following investigations.

As one can see that there are big differences in the  $k_{\parallel} - k_{\perp}$  spectrum of both the electrostatic and electromagnetic waves between the inhomogeneous-density and inhomogeneous-temperature plasmas. For the electrostatic waves, their energies are mainly located at  $k_{\parallel} < 3\omega_{\text{norm}}/c$  ( $k_{\parallel} > 2\omega_{\text{norm}}/c$ ) along the quasi-parallel direction to the background magnetic field in the  $k_{\parallel} - k_{\perp}$  space for the inhomogeneous-density (inhomogeneous-temperature) plasma. The strongly excited electromagnetic waves quasi-perpendicular propagate in both plasmas, but the bandwidth of these electromagnetic waves have wider spreads in the  $k_{\parallel} - k_{\perp}$  space for plasma with inhomogeneous density than those in the inhomogeneous-temperature plasma. These differences indicate that the excited (both electromagnetic and electrostatic) waves might have different properties between the inhomogeneous-density and inhomogeneous-temperature plasmas although free energy provided by energetic ring-beam electrons are the same in these inhomogeneous plasmas. In the following, we will investigate the properties of these excited waves, e.g., mode of these these excited waves, polarization.

### 3.2.1. Electromagnetic Waves

Figs. 4 and 5 present the dispersion spectrum of the electromagnetic waves propagating along different directions over the whole simulation domain. Polarization property of these electromagnetic waves are shown in the columns (a1), (a2) and (a3) of Figs. 6 and 7. Spectra of the dispersion relation (Figs. 4 and 5) together with the polarization (Figs. 6 and 7) can help us to figure out which wave mode got stronger excitation. Consistent with Figs. 2 and 3, one can also find that energy of the electromagnetic waves reaches its peak after  $\omega_{\text{norm}}t = 180$  and that more strongly excited electromagnetic waves quasi-perpendicular propagate in both plasmas with Figs. 4, 5, 6 and 7.

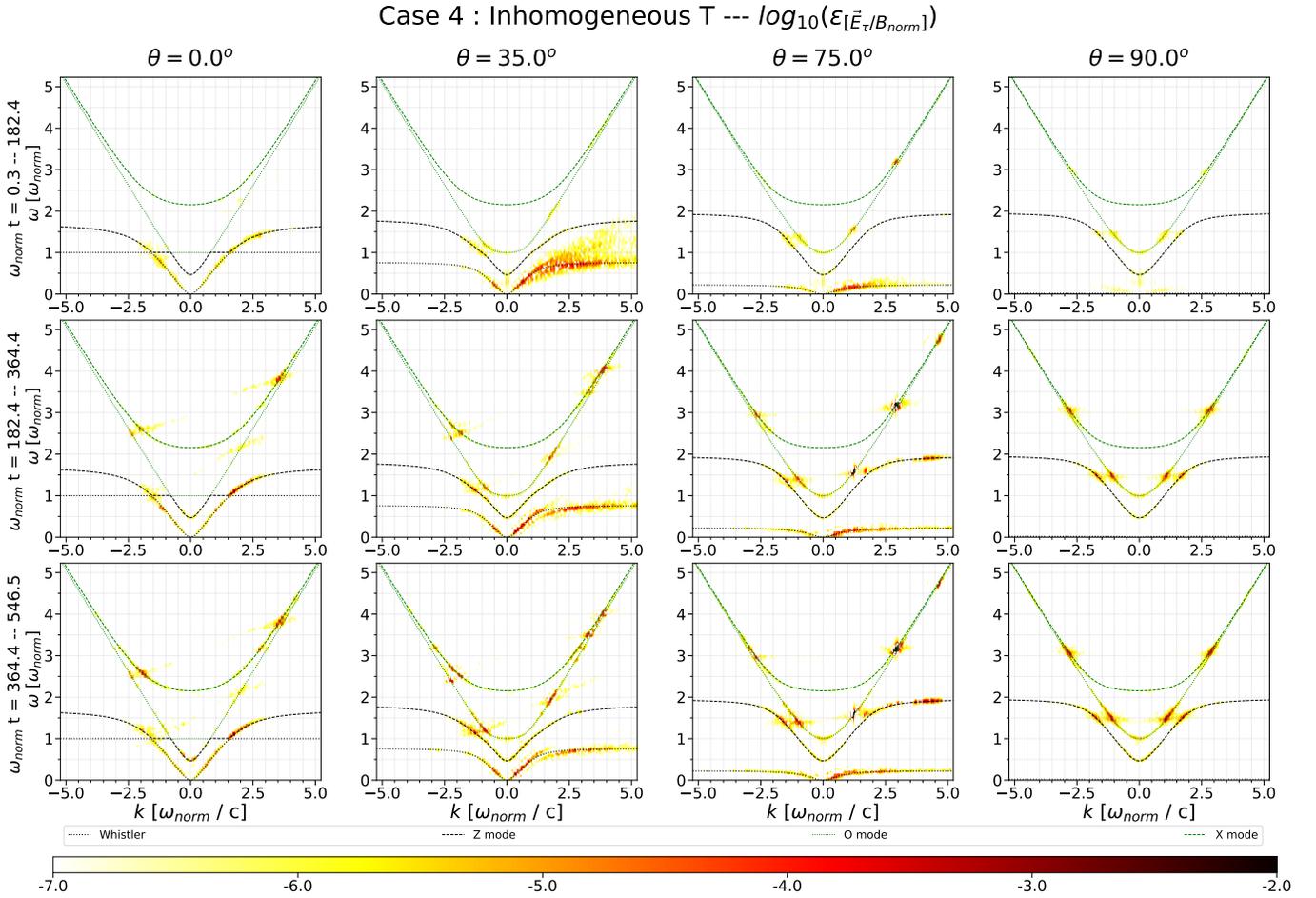
When  $\omega_{\text{norm}}t < 182$ , both the the beam and ECM instabilities could be triggered (since  $\omega_{\text{norm}}t = 100$ ) processes play roles in the excitation of the electromagnetic waves in the inhomogeneous-density plasma, see panel (b1) of Fig.2. Correspondingly, the more strongly excited electromagnetic waves are the right-handed polarized whistler (with  $\omega/\omega_{\text{norm}} < 1$ ) and Doppler-shifted Bernstein or fundamental X-mode, i.e., X1-mode (Pritchett 1984 with  $\omega/\omega_{\text{norm}} \sim \omega_{ce, \text{mean}}$ ) waves, see top panels in Fig. 4 and in columns (a1) to (a3) of Fig. 6.  $\omega_{ce, \text{mean}}$  is the mean value of the electron gyrofrequency  $\omega_{ce}$  over the whole simulation domain and  $\omega_{ce, \text{mean}} = \overline{\omega_{ce}} = 1.69\omega_{\text{norm}}$ , see panel (e) of Fig. 1. Due to evolution of the parallel motion of the energetic ring-beam electrons, these excited Doppler-shifted X1-mode waves cover a quite wide frequency range especially in the direction along the background magnetic field, see the blue dashed lines covered range in the top panels of Fig.4. Meanwhile energy of the right-handed polarized second harmonic X-mode waves, i.e., X2-mode (with  $\omega \sim 2\omega_{ce, \text{mean}}$  and  $c|k|/\omega_{\text{norm}} \sim 3$ ) slightly increase also along the quasi-perpendicular directions during this time period. In general, for the inhomogeneous-density plasma, the X1-mode waves dominate the energy of all excited electromagnetic waves. And these excited X1-mode waves can be detected almost over the whole propagating angle  $\theta$  space, while the excited whistler and X2-mode waves propagate mainly with propagating angle  $\theta \sim 35^\circ$  and  $50^\circ < \theta < 90^\circ$ , respectively. In the inhomogeneous-temperature plasma, there is, however, no obvious excitation for the X1-mode waves, and instead, most of energy of the excited electromagnetic waves are located in the whistler waves, see top row of Fig. 5. Additionally a weak excitation signal for the left-handed polarized O1-mode waves can be found in the region of  $1.5 < \omega/\omega_{\text{norm}} < 2$ ,  $c|k|/\omega_{\text{norm}} \sim 1$ , and  $\theta \sim 60^\circ$ , where one could detect a wave with the left-handed polarization, see top



**Figure 4.**  $\vec{k} - \omega$  (or dispersion relation) power spectrum of the transverse electric field  $E_\tau$  for the electromagnetic waves propagating along different directions  $\theta = |\arctan(k_\perp/k_\parallel)|$  (different columns) over the entire space domain but three different time periods (different rows) in the inhomogeneous-density plasma (i.e., Case 2). In these panels,  $|\vec{k}| = \sqrt{k_\parallel^2 + k_\perp^2}$  and the sign of  $k$  is the same as its parallel component  $k_\parallel$ . While the overplotted lines are the four magnetoionic modes in the magnetized cold plasma limit (Willes & Cairns 2000), from bottom to top, they are the whistler (black dotted lines), Z (black dashed lines), O (green dotted lines) and X (green dashed lines) modes, respectively, where the mean value of  $\omega_{ce}$  over the whole simulation domain, i.e.  $\omega_{ce,mean}/\omega_{norm} = 1.69$  and  $\omega_{norm}$  for the plasma frequency are applied to the dispersion relation of these four magnetoionic modes. Moreover, blue dashed lines around  $\omega_{ce,mean}$  in the top panels are the Doppler-shifted Bernstein mode with  $\omega = \omega_{ce,mean}/\gamma_{d,erb} + \vec{k} \cdot \vec{U}_{d,erb}$ , where  $U_{d,erb}$  is the parallel drift velocity of energetic electrons.  $U_{d,erb}$  could range from 0 to  $0.47c$  and the Lorentz factor of energetic electrons  $\gamma_{d,erb} = 1.0$  to  $1.2$ .

panels in columns (a1) to (a3) of Fig. 7. Excitation of the whistler and O1-mode waves in the inhomogeneous-temperature plasma could be due to the beam instability (i.e., the plasma emission process) only since there is still no obvious energy decrease in the perpendicular momentum profile of the energetic ring-beam electrons before  $\omega_{norm}t = 200$ .

During the time period  $\omega_{norm}t = 182 - 364$ , when excitation of the electromagnetic waves is taken over by the ECM instability (see Sec. 3.1), the fundamental waves (with  $\omega \sim \omega_{ce,mean}$ , i.e., the X1 waves, the right-handed polarized Z-mode, and the left-handed polarized O-mode waves) as well as the second (with  $\omega \sim 2\omega_{ce,mean}$ ), and third (with  $\omega \sim 3\omega_{ce,mean}$ ) harmonic waves get energy enhanced in both plasmas, see the middle panel in columns (a1) to (a4) of Figs. 6 and 7 together with the middle row of Figs. 4, 5, respectively. Frequency of the fundamental and high order harmonic electromagnetic waves are, hence, related to  $\omega_{ce,mean}$  in this study. Among these excited electromagnetic waves, the more strongly excited ones are located in the second harmonic branch and propagate mainly quasi-perpendicular with  $50^\circ < \theta < 90^\circ$  (see the middle panel in columns a1, a2 of Figs. 6 and 7). Due to the energy enhancement of the fundamental O-mode waves, there are a few large regions dominated by the left-handed polarized electromagnetic waves in the  $\theta - \omega$  space in both plasmas, e.g., ( $30^\circ < \theta < 50^\circ$ ,  $1.5 < \omega/\omega_{norm} < 2$ ) in the inhomogeneous-density plasma and ( $20^\circ < \theta < 90^\circ$ ,  $1.5 < \omega/\omega_{norm} < 2$ ) in the inhomogeneous-temperature plasma, see the

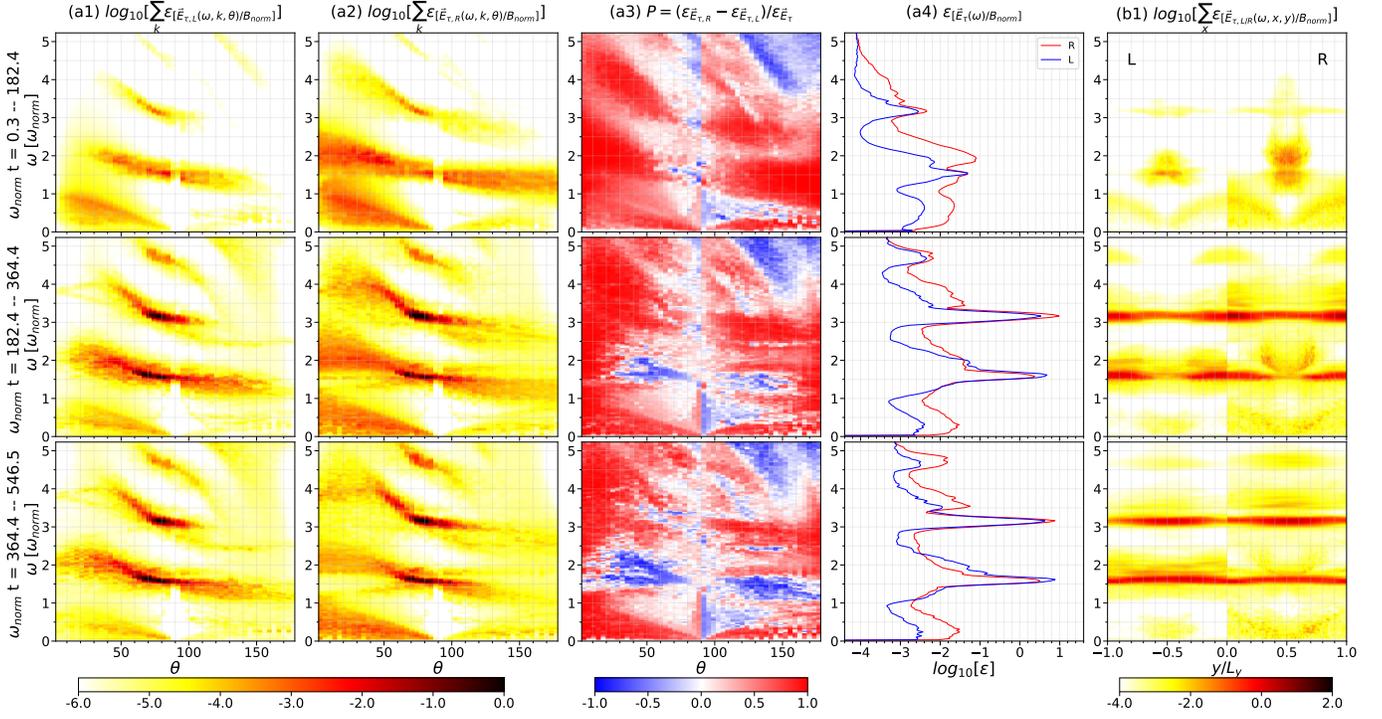


**Figure 5.** Similar to Fig. 4, but for the plasma with inhomogeneous temperature (i.e., Case 4).

middle panel in columns (a3) of Figs. 6 and 7, respectively. Beside of the above mentioned regions, there is another left-handed polarization dominant region ( $120^\circ < \theta < 170^\circ$ ,  $\omega/\omega_{norm} \sim 2.5$ ) located at the second harmonic branch in the inhomogeneous-temperature plasma, even though the backward propagating O-mode waves in this region (see the first and second panels in the second row of Fig. 5) are not so strongly excited as those forward propagating O-mode waves. Overall, in both plasmas, energy of the fundamental waves is dominated by the O-mode waves and a left-handed polarization could mostly be detected, while the right-handed polarized X-mode waves are in charge of the properties of the second and third harmonic waves (see the middle panel in the column a4 of Figs. 6 and 7). Considering a remote detection, left-handed polarization degree of the fundamental branch could be even larger than that in their source region, since the right-handed polarized Z-mode waves in the fundamental branch could not escape from their source region if these Z-mode waves do not experience wave mode conversion to escaping electromagnetic waves.

After  $\omega_{norm} t = 364$ , when excitation of the electromagnetic waves has been reached their saturation (see panel a2 of Fig. 2), there is no excitation of a new mode wave, and polarization properties of different harmonic branches are similar as those in the earlier time period. Energy partition among these excited electromagnetic waves, however, changes in each kind of plasma. For instance, energy of the second harmonic right-handed polarized waves decreases along with a narrower and narrower peak band around  $2\omega_{ce,mean}$ , while the fundamental and third harmonic waves gain energies after  $\omega_{norm} t \sim 364$ , see the middle and bottom panels in the column (a4) of Figs. 6 and 7. In the inhomogeneous-density plasma, energy of the left-handed polarized fundamental waves even reach a similar level to that of the second harmonic waves after  $\omega_{norm} t = 364$ . There is, hence, a possibility that the second harmonic waves (2H) might decay into the fundamental (F) and third (3H) harmonic waves, i.e.,  $2H + 2H \rightarrow F + 3H$  during the nonlinear period of the wave excitation in both plasmas. Due to the energy enhancement of the O-mode waves in the

## Case 2 : Inhomogeneous n

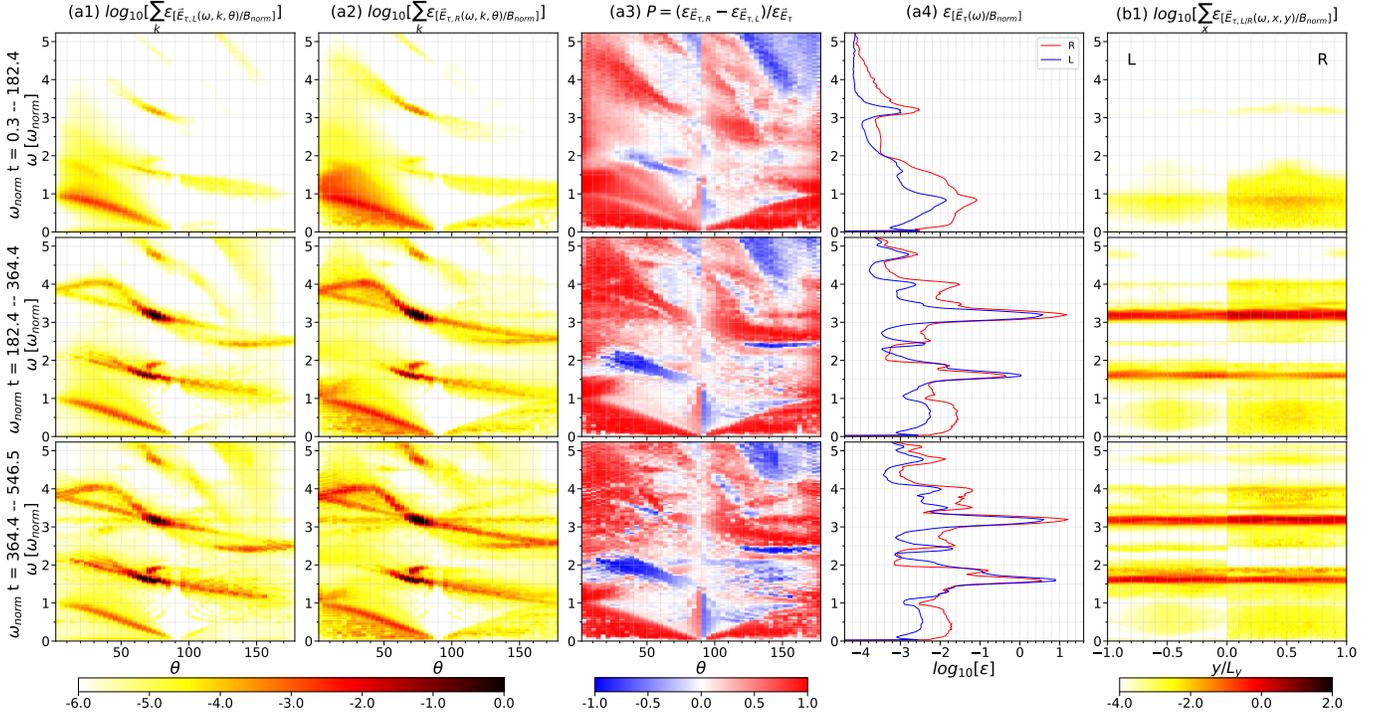


**Figure 6.** For the inhomogeneous-density plasma, columns (a1) and (a2) show the energy distribution for the left-handed ( $\epsilon_{\vec{e}_{\tau,L}}$ ) and right-handed ( $\epsilon_{\vec{e}_{\tau,R}}$ ) polarized electric field of electromagnetic waves in the propagating angle-frequency ( $\theta - \omega$ ) space, respectively. Polarization degree ( $P = (\epsilon_{\vec{e}_{\tau,R}} - \epsilon_{\vec{e}_{\tau,L}}) / \epsilon_{\vec{e}_{\tau}}$ , where  $\epsilon_{\vec{e}_{\tau}} = \epsilon_{\vec{e}_{\tau,R}} + \epsilon_{\vec{e}_{\tau,L}}$ ) of the electromagnetic waves in the whole simulation domain is presented in column (a3). Column (a4) displays the energy distribution for the right-handed (red solid line) and left-handed (blue solid line) polarized electric field of electromagnetic waves in the frequency  $\omega$  space. Column (b1) exhibits the energy distribution for the left-handed (right-handed) polarized electric field of electromagnetic waves in the  $y - \omega$  space with  $y/L_y < 0$  ( $y/L_y > 0$ ). Different rows are used to present evolution of the above-mentioned parameters over three different time periods.

fundamental branch during this time period, the left-handed polarization dominated regions get enlarged and enhanced around  $\omega_{ce,mean}$  in the  $\theta - \omega$  space in both plasmas, see the middle and bottom panels in column (a3) of Figs. 6 and 7.

Additionally, note that there are different energy evolutions for the low frequency ( $\omega/\omega_{norm} < 1$ ) whistler waves in these two kinds of inhomogeneous plasmas. In the inhomogeneous-density plasma, peak energy of the whistler waves increases and moves to lower frequencies with the evolution of the plasma system, accompanied by the energy decrease of the X1-mode waves (see Fig. 4 and column a4 of Fig. 6). The X1-mode waves might, hence, contribute to the energy gain of the whistler waves via wave-wave interaction. On the contrary, total energy of the whistler waves always decreases after its excitation in the inhomogeneous-temperature plasma (Case 4), where excitation of the X1-mode waves is missing (see Fig. 5 and column a4 of Fig. 7). Moreover, one can also find that, in the  $\theta - \omega$  space with the increase of the wave propagating angle  $\theta$ , there is a frequency drift from higher to lower frequencies in each harmonic band for both inhomogeneous plasmas (columns a1 and a2 of Figs. 6 and 7). Similar to the excited Doppler-shifted X1-mode in the inhomogeneous-density plasma, this frequency drift might also be due to the Doppler effect due to the parallel drifting motion of the energetic ring-beam electrons, i.e.,  $\omega = h\omega_{ce,mean} / \gamma_{d,erb} + k_{\parallel} u_{d,erb,\parallel} \cos \theta$ , where  $h$  is the harmonic index. A smaller wave propagating angle  $\theta$  could lead to a larger frequency shift from the characteristic frequency  $h\omega_{ce,mean}$  of the  $h$ -harmonic branch. As a result, frequency separation between adjacent harmonic branches will be influenced by the wave propagating angle  $\theta$ , property of the dominant electromagnetic waves as well as harmonic index  $h$ . Equidistant frequency separation, hence, more likely appear among harmonic branches with larger harmonic index  $h$ . Last but not the least, these excited electromagnetic waves have wider frequency bandwidths in the inhomogeneous-density plasma than those in the inhomogeneous-temperature plasma (columns a1 and a2 of Figs. 6 and 7), which coincides with the theoretical study of [Winglee & Dulk 1986](#). Correspondingly spectrum of the excited electromagnetic waves has a more wider spread over the  $k_{\parallel} - k_{\perp}$  spaces in the inhomogeneous-density plasma (see panels b1 and b2 of Fig. 3).

## Case 4 : Inhomogeneous T



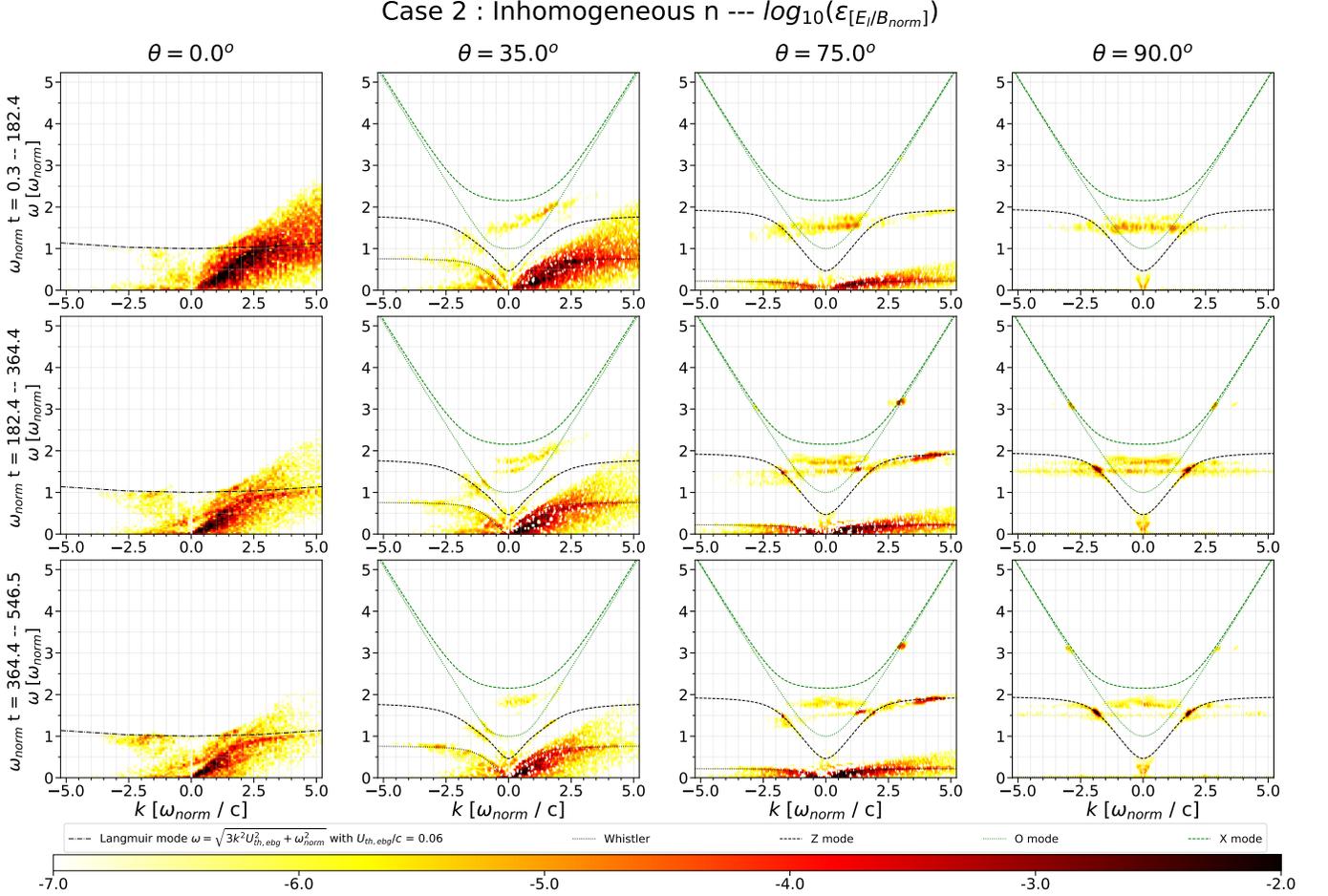
**Figure 7.** Same as Fig. 6, but for the plasma with inhomogeneous temperature (i.e., Case 4).

Column (b1) of Figs. 6 and 7 exhibits the energy distribution for both the left-handed and right-handed polarized electric field of electromagnetic waves in the  $y - \omega$  space over different time periods. In order to separate the left-handed and right-handed polarized waves, electric energy of these left-handed polarized waves are present with  $y/L_y < 0$ , their locations along the  $y$ -axis of the simulation domain are equal to  $|y|$ . One could, hence, figure out the excitation location (or source region) of these excited electromagnetic waves with these panels.

In the inhomogeneous-density plasma (Case 2), before  $\omega_{norm}t \sim 182$ , the right-handed elliptically polarized X1-mode waves with  $\omega/\omega_{norm} \sim 2$  are excited around the center of the  $y$ -axis, where has a quite tenuous background plasma (see panel b of Fig. 1). During the same period, the whistler waves with  $\omega/\omega_{norm} < 1$  have more energies located further away from the central  $y$ -axis. In addition, due to the dispersion relation of the whistler wave depending on the local plasma frequency  $\omega_{pe}$  (for instance, the resonance frequency of the whistler wave is  $\sqrt{(\omega_{pe}^2 + \omega_{ce}^2 - \sqrt{(\omega_{pe}^2 + \omega_{ce}^2)^2 - 4\omega_{pe}^2\omega_{ce}^2\cos^2\theta})}/2$  Willes & Cairns 2000), these excited whistler wave cover a wider and wider frequency range from the center to the boundary of the  $y$ -axis, i.e.,  $|y|/L_y = 0$  to 1. During the second growth period of the electromagnetic waves, i.e.,  $\omega_{norm}t = 182 \sim 364$ , energy of those stronger excited (fundamental, second, and third) electromagnetic harmonic waves are also mainly located at the boundaries of the  $y$ -axis. During the nonlinear period  $\omega_{norm}t > 364$ , most of these stronger excited electromagnetic waves emplace their energy more around the center of the  $y$ -axis due to the propagation of these electromagnetic waves as well as weaker wave damping there with a low plasma density. Similar location between the energy-decreased X1-mode and energy-enhanced whistler waves at the end of simulation indicates again that the X1-mode waves might contribute to the energy gain of the whistler waves via wave-wave interactions.

In the inhomogeneous-temperature plasma (Case 4), different from those in the inhomogeneous-density plasma, there is no excitation signal for the X1-mode waves, and furthermore energy of all excited electromagnetic waves are more or less homogeneously distributed along the  $y$ -axis, similar to the previous related studies in homogeneous plasmas.

Location of the X1-mode source region with low plasma density in the inhomogeneous-density plasma as well as excitation absence of the X1-mode waves in the inhomogeneous-temperature plasma with denser background plasma indicate that excitation of the X1-mode waves prefer to occur in low density plasmas with a larger frequency ratio of  $\omega_{ce}/\omega_{pe}$ . High-density source region for other excited electromagnetic (e.g., Z, harmonics of the O, higher harmonic X-mode) waves implies that stronger excitation



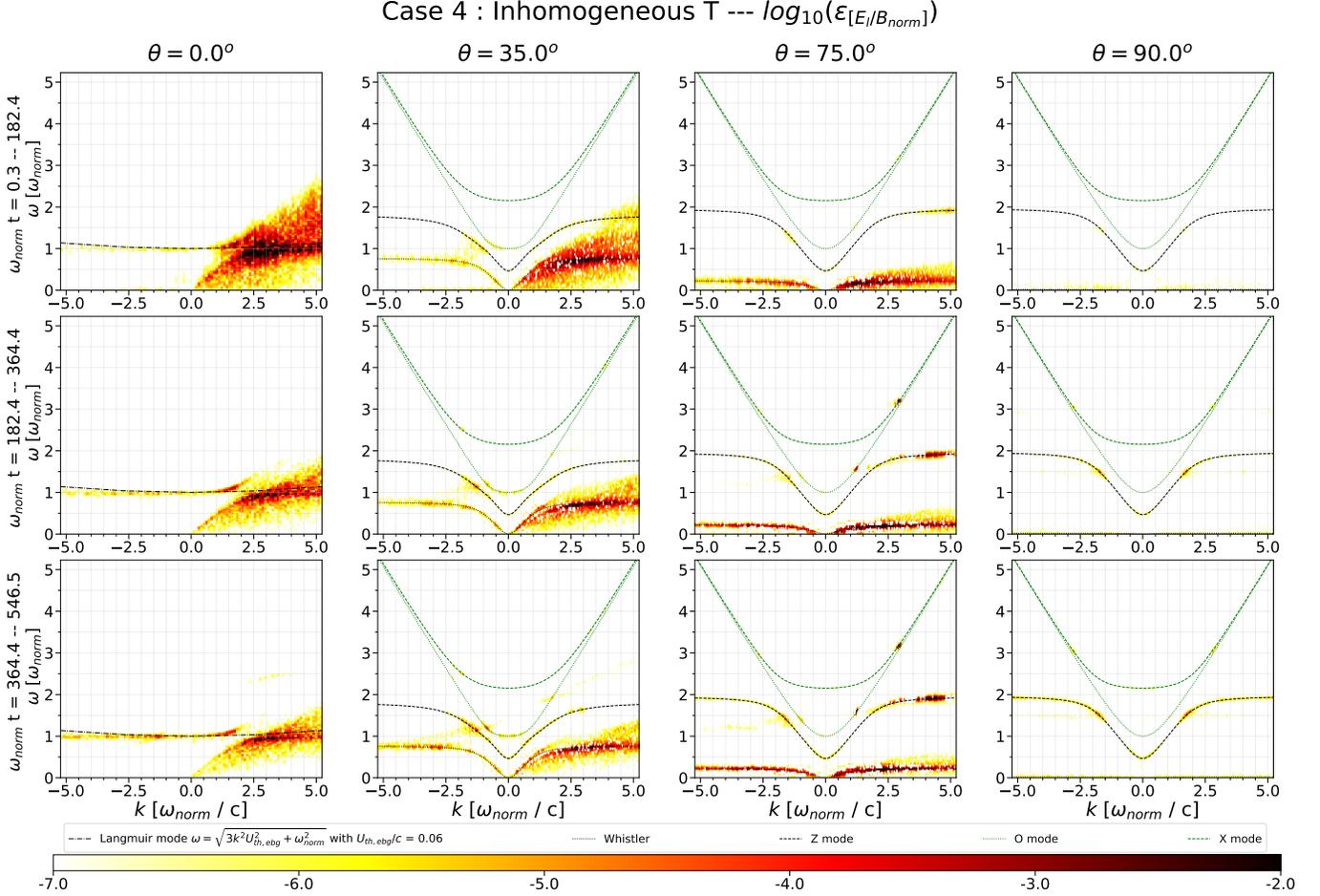
**Figure 8.** Similar to Fig. 4, but for the longitudinal electric field  $E_{\parallel}$  in the inhomogeneous-density plasma (i.e., Case 2). In each panel of the left-hand first column, the overplotted dash-dot line presents the dispersion relations of the Langmuir wave ( $\omega = \sqrt{3k^2 U_{th,ebg}^2 + \omega_{norm}^2}$ ). The overplotted line in other panels are the same with that in Fig. 4.

of these waves happen in plasmas with a relative smaller frequency ratio of  $\omega_{ce}/\omega_{pe}$ . These conclusions are, actually, consistent with the analytical investigations for the growth rate of different wave modes excited by the ECM instability, e.g., [Tong et al. 2017](#); [Chen et al. 2017](#); [Zhao et al. 2016](#). In these investigations, they found that the maximum growth rate of the X1-mode (other electromagnetic mode) waves located at a region with larger (smaller) frequency ratio of  $\omega_{ce}/\omega_{pe}$  in spite of the momentum distribution of the energetic electrons. Additionally, different source regions of the excited waves between the inhomogeneous-density and inhomogeneous-temperature plasmas indicate that density of the background plasma has a stronger influence on wave excitation than its temperature. That might be the reason of neglecting the temperature effect on the growth rate of different wave modes in the above mentioned analytical investigations for (non-relativistic) space plasmas.

### 3.2.2. Electrostatic Waves

Figs. 8, 9 and 10 present energy distribution of the longitudinal electric field  $E_{\parallel}$  in the dispersion ( $k - \omega$ ), propagating angle-frequency ( $\theta - \omega$ ) as well as y-axis-frequency spaces. Longitudinal electric field  $E_{\parallel}$  can, to some extent, exhibits properties of electrostatic waves although some electromagnetic waves contain longitudinal electric field also, e.g., the whistler, Z and X-mode waves ([Willes & Cairns 2000](#); [Huang & Lyu 2019](#)).

As we know that two kinds of electrostatic waves can be excited in the frequency range of  $\omega \gg \omega_{ci}$  in the presence of an electron beam in plasma, i.e., the Langmuir wave and the beam wave  $\omega = k_{\parallel} v_b$ , where  $\omega_{ci}$  and  $v_b$  are the gyrofrequency of protons and the bulk velocity of the energetic electron beam, respectively ([Gary 1993](#)). With Figs. 8, 9 and columns (a1) and (a2) of Fig. 10, one can find that most energy of these excited electrostatic waves are located in those quasiparallel (in particular parallel) propagating ones with  $\omega/\omega_{norm} < 2$ , and we consider that these parallel propagating and strongly excited electrostatic waves

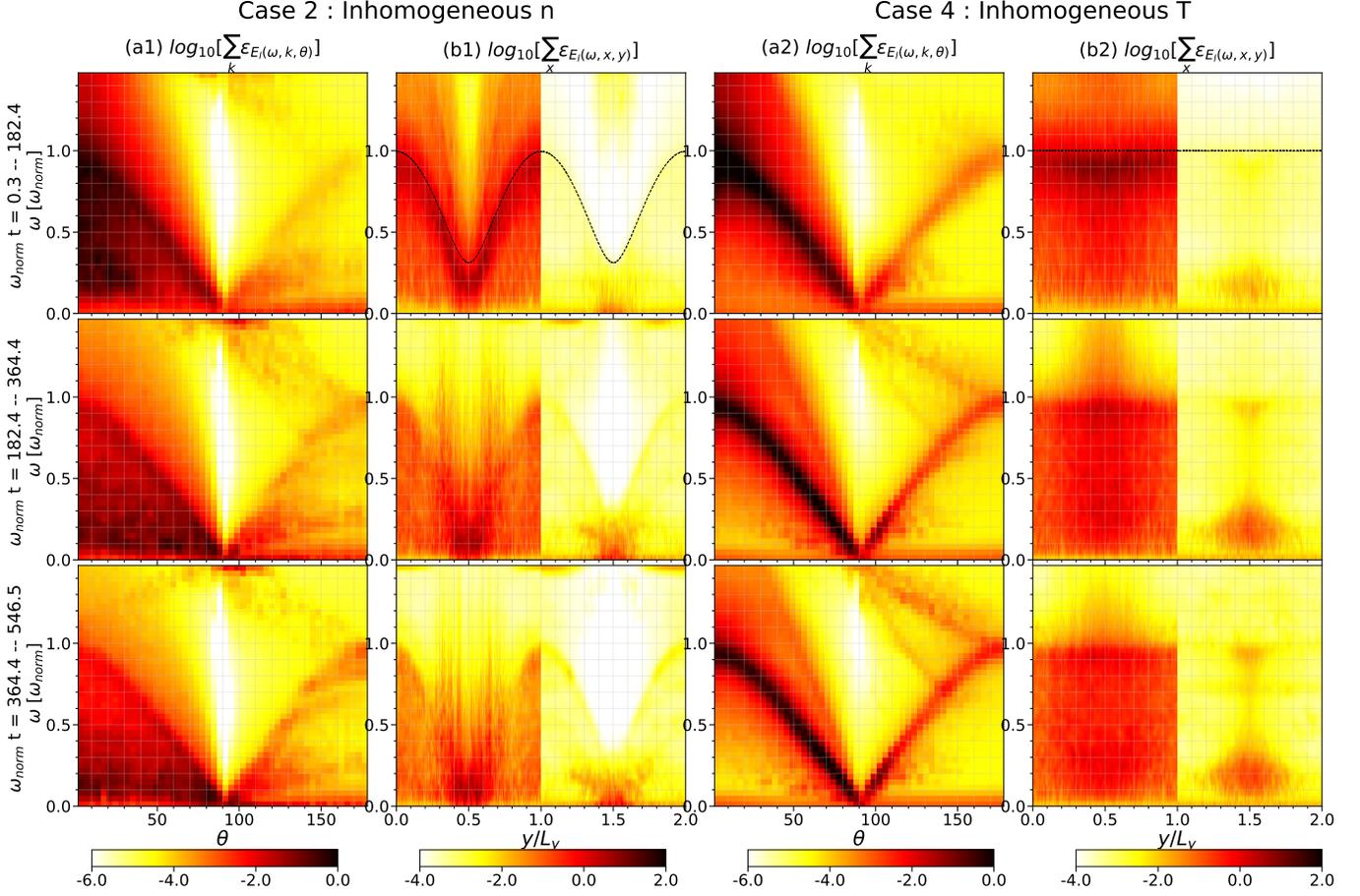


**Figure 9.** Same as Fig. 8, but for the plasma with inhomogeneous temperature (i.e., Case 4).

are the Langmuir wave in both plasmas. That can be easily identified in the inhomogeneous-temperature plasma (Case 4) with uniform background electron density and plasma frequency (see panels b and d of Fig. 1), where one can find that the mostly excited parallel propagating electrostatic waves are located near the uniform Langmuir wave in the dispersion relation space (see the left-hand first column of Fig. 9). In the inhomogeneous-density plasma (Case 2), with top panel in column (b1) of Fig. 10, one can find that frequency of these stronger excited electrostatic waves also have similar y-axis dependence to the background inhomogeneous plasma frequency. The beam-mode wave would be, however, homogeneously distributed along the y-axis if the corresponding energetic electron beams are uniformly distributed in plasma, which we apply in this study. Due to the wider range of the plasma frequency  $\omega_{pe}$  in the inhomogeneous-density plasma, spectrum of the excited electrostatic Langmuir wave have a more wider spread in the  $k - \omega$  (see the left-hand column of Fig. 8) as well as the  $k_{||} - k_{\perp}$  spaces (see panel a1 of Fig. 3).

With the increase of the propagating angle  $\theta$ , in both inhomogeneous plasmas, the dispersion relation of these more energized longitudinal electric fields  $E_l$  coincide well with the dispersion relation of the magnetoionic modes in the magnetized cold plasma limit as well as the transverse electric fields  $E_t$  of those excited electromagnetic waves (i.e., Figs. 4 and 5). That indicates that the longitudinal electric fields  $E_l$  with larger  $\theta$  are the longitudinal part of those excited electromagnetic waves, and these excited (pure) electrostatic waves are mainly the Langmuir waves with  $\omega/\omega_{norm} \leq 1$  propagating quasiparallel to the background magnetic field. In the followings, we, hence, consider only the  $E_l$  with  $\omega/\omega_{norm} < 1.5$  for electrostatic Langmuir waves in both inhomogeneous plasmas in Fig. 10, where the forward (with  $k_{||} > 0$ ) and backward (with  $k_{||} < 0$ ) propagating electrostatic waves are separated with ( $\theta < 90^\circ$ ,  $\theta > 90^\circ$ ) in columns (a1, a2) and ( $y/L_y < 1$ ,  $y/L_y > 1$ ) in columns (b1, b2), respectively.

As time goes on, these strongly excited forward (quasiparallel) propagating Langmuir waves, however, dissipate faster than the longitudinal electric fields  $E_l$  of those quasiperpendicular propagating electromagnetic waves, see Figs. 8 and 9, probably due to the strong Landau damping processes of Langmuir waves. And this dissipation mainly occurs at the y-axis boundary of



**Figure 10.** Energy distribution of the longitudinal electric field  $E_l$  in the propagating angle-frequency ( $\theta - \omega$ , presented in columns a1, a2) as well as y-axis-frequency spaces (presented in columns b1, b2) for the inhomogeneous-density (Case 2) and inhomogeneous-temperature (Case 4) plasmas, respectively. In columns (b1) and (b2), region with  $y/L_y < 1$  ( $y/L_y > 1$ ) is for the longitudinal electric field  $E_l$  propagating with  $k_{\parallel} > 0$  ( $k_{\parallel} < 0$ ). And the overplotted dashed line in the top panel of columns (b1) and (b2) present the localized plasma frequency  $\omega_{pe}$  along the y-axis, same with panel (d) in Fig. 1. While different rows are used to present evolution of the above-mentioned parameters over three different time periods.

the simulation domain in both plasmas (see the region with  $y/L_y < 1$  in columns b1 and b2 of Fig. 10), where the larger density and/or higher temperature of the background electrons (than those around the center of the simulation domain, see panels a and b of Fig. 1) benefit the Landau damping process. While with the energy decrease of these strongly excited forward propagating Langmuir waves, energy of their corresponding backward propagating parts increase, see the left column of Figs. 8 and 9 as well as columns (a1) and (a2) of Fig. 10. That implies that those backward propagating Langmuir waves probably come from scattering of these forward propagating Langmuir waves. Those backward propagating Langmuir waves, however, have different locations along the y-axis in the inhomogeneous-density and inhomogeneous-temperature plasmas. In the inhomogeneous-density plasma, frequency of the Langmuir waves vary along y-axis but the scattered backward propagating Langmuir waves are mainly located around  $\omega/\omega_{norm} \sim 1$  (see the bottom panel in the left column of Fig. 8) corresponding to the plasma frequency at the y-axis boundary of the simulation domain. Most energy of these backward propagating Langmuir waves with  $\omega/\omega_{norm} \sim 1$  are indeed located at the y-axis boundary, see the region with  $y/L_y > 1$  in column b1 of Fig. 10, where the larger density of the background particles would enhance scattering of Langmuir waves. On the contrary, the backward propagating Langmuir waves in the inhomogeneous-temperature plasma are mainly located at the center of the plasma system (see the region with  $y/L_y > 1$  in column b2 of Fig. 10) due to the weaker dissipation of the forward propagating Langmuir waves there. Therefore, in total, energy dissipation of these forward propagating Langmuir waves in plasmas are more likely due to both the Landau damping and the wave scattering processes.

In the plasma emission mechanism, as we know, scattering of Langmuir waves can lead to the enhancement of the backward propagating Langmuir waves as well as the left-handed polarized electromagnetic waves with  $\omega \sim \omega_{pe,loc}$  (where  $\omega_{pe,loc}$  is the local plasma frequency). Meanwhile there could be enhancement of the left-handed polarized quasiparallel electromagnetic waves with  $\omega \sim 2\omega_{pe,loc}$  in the presence of both the forward ( $L$ ) and backward ( $L'$ ) quasiparallel propagating Langmuir waves, i.e.,  $L + L' \rightarrow 2H_p$ . There are, indeed, energy enhancement of electromagnetic waves along the quasiparallel directions with  $\omega \sim \omega_{norm}$  as well as  $\omega \sim 2\omega_{norm}$  in both plasmas (see column a1 in Figs. 6 and 7), whether the process of  $L + L' \rightarrow 2H_p$  contributes to this energy enhancement, however, still needs further investigations in our future studies. But it is certain that electromagnetic waves with  $\omega \sim \omega_{norm}$  and  $\omega \sim 2\omega_{norm}$  along the quasiparallel directions do not contribute to the energy peak of the left-handed polarized electromagnetic waves, which are located at  $\omega \sim \omega_{ce,mean}$ , and propagating along the quasiperpendicular directions.

#### 4. CONCLUSIONS AND DISCUSSION

In this paper we focus on studying the coherent emission mechanism behind the solar radio bursts. Coherent solar radio emissions assuming homogeneous plasmas conditions at their source region have been widely studied via theoretical analysis or/and simulations. There is, however, seldom plasma immersed in a uniform magnetic field in realistic solar coronal environment. Inhomogeneities exist over almost all spatial scales of the solar corona. As it is known, diverse macroscopic phenomena (e.g., convection, thermal conduction) can generate inhomogeneity at large scales and many of them could cascade down to kinetic scales, in particular via turbulence, which is ubiquitous in the solar corona. Turbulence can also, in particular, be generated by processes associated to solar flares and radio emission, like magnetic reconnection and shocks. As mentioned by [Melrose 1975](#) and [Winglee & Dulk 1986](#), the inhomogeneous nature of plasmas is quite important as it influences properties of the emission spectrum of radio bursts. Contribution of the inhomogeneity of background magnetic field as well as density and/or temperature in plasmas on the properties of the coherent radio emission for solar radio bursts, hence, needs to be further verified and expanded. In this paper, we report the excitation properties of both the electromagnetic and the electrostatic waves produced by ring-beam distributed energetic electrons in inhomogeneous magnetized equilibrium plasmas of solar corona via selfconsistent 2.5-dimensional particle-in-cell (PIC) code numerical simulations. The disequilibrium introduced by the inhomogeneous background magnetic field is, we consider, balanced by either inhomogeneous density or inhomogeneous temperature of the background plasma, corresponding to inhomogeneous-density plasma and inhomogeneous-temperature plasma, respectively. And inhomogeneity of these plasmas exist only along the dimension perpendicular to the background magnetic field in this study.

Same with homogeneous plasmas, the ring-beam distributed energetic electrons can excite waves associated to the two most promising mechanisms for the coherent solar radio emission: the beam and ECM instabilities. Those instabilities can excite electromagnetic waves for radio emissions. These excitations occur due to the free energy (i.e., the positive gradients) along the parallel and perpendicular directions to the ambient magnetic field in the energetic ring-beam electron velocity distribution, respectively. However properties of these excited waves are not always the same between the inhomogeneous-density and inhomogeneous-temperature plasmas. For instance,

**Similarities:** The onset of the ECM instability is later than the beam instability. The beam instability mainly excite electrostatic Langmuir waves propagating quasiparallel to the background magnetic field as well as the electromagnetic whistler waves. The ECM instability, however, fully contributes to the excitation of electromagnetic Z, O, and X-mode waves. Saturation energy of these excited electrostatic and electromagnetic waves have the same order of magnitude in these two inhomogeneous plasmas. For the electromagnetic waves, these stronger excited ones mainly propagate quasiperpendicular to the background magnetic field, and most of their energies are located around  $h\omega_{ce,mean}$ , where  $\omega_{ce,mean}$  is the mean value of the electron gyrofrequency over the whole simulation domain and  $h$  is the harmonic index. The fundamental branch presents a left-handed polarization due to the energy dominance of the O-mode waves, while the X-mode waves determine the properties of the second and third harmonic branches. The low frequency whistler waves are more likely found in region with a low plasma density and low temperature, which could reduce wave damping. The backward quasiparallel Langmuir waves can be detected after the excitation of the forward Langmuir waves.

**Differences:** **In the inhomogeneous-density plasma**, there is an excitation of the right-handed polarized X1-mode waves contributed by the ECM instability. During the excitation period, most energy of the X1-mode (whistler, Z, harmonics of the O, higher harmonics of the X-mode as well as the backward Langmuir) waves are located around the region with a tenuous (dense) background plasma. During the nonlinear period, most of these excited electromagnetic waves emplace their energy more in the tenuous-plasma region, where the low plasma density could reduce the damping of these excited electromagnetic waves. **In the inhomogeneous-temperature plasma**, excitation of the X1-mode wave is absent. Other excited electromagnetic waves are more homogeneously distributed along the inhomogeneity gradient over the whole wave

evolution. There are also more energized backward Langmuir waves than those in the inhomogeneous-density plasma. These backward Langmuir waves could be detected only around the region with a low temperature and a high density as the dense region of the inhomogeneous-density plasma. Additionally, frequency bandwidth of different harmonic branches are thinner than those in the inhomogeneous-density plasma.

As prediction by those analytical investigations for the growth rate of different wave modes excited by the ECM instability (e.g., Tong et al. 2017; Chen et al. 2017; Zhao et al. 2016), different source regions between the X1-mode and other electromagnetic modes in the inhomogeneous-density plasma are due to their excitations strongly depending on the frequency ratio  $\omega_{pe}/\omega_{ce}$ . Temperature of the background plasma does, however, not influence much on the excitation of the electromagnetic waves in non-relativistic space plasmas. Wave excitation properties in the inhomogeneous-temperature plasma are indeed similar to those in homogeneous plasmas known from previous related investigations for homogeneous plasmas.

Based on the properties of the excited electromagnetic waves in this paper, our results can be applied to explain some features of the solar radio bursts with zebra-stripe pattern, e.g., lace burst (Karlický et al. 2001), fiber burst (Chernov et al. 2014), zebra-pattern burst (Huang & Tan 2012; Chernov 2015) as well as narrowband spikes (Krucker & Benz 1994; Karlický et al. 2021, 2022):

**Slow frequency drifting rate:** Most of the solar radio bursts with zebra-stripe pattern often exhibit slow frequency drifting rates compared to those of the solar type III radio burst (Aschwanden 2005; Tan et al. 2014; Chernov 2015). That indicates that source of these radio bursts might be slow electron beams. Based on our results, the earlier state of the related slow electron beams could drift quite fast while they slow down due to the onset of the beam instability. The beam instability could not always lead to efficient wave excitation for radio emissions (Thurgood & Tsiklauri 2015). Slow-down electron beams with population inversion along the direction perpendicular to the background magnetic field (e.g., for the ECM instability) are more likely to play an important role in the generation of those solar radio bursts with slow frequency driftings.

**Zebra-structure stripes:** Our results show that, in dependence on the wave propagating angles  $\theta$ , one or more harmonic bands can escape from their source region, which could potentially explain the detection of various number of harmonic branches in radio bursts with zebra-stripe pattern. Also, variation of frequency bandwidth in single stripe of radio bursts with zebra-stripe pattern might be related to variation of the perpendicular plasma density gradient in the source region of the stripes. Based on our results, a larger perpendicular density gradient in plasmas could lead to a wider frequency bandwidth in each harmonic branch. Note that, according to the study of Yao et al. 2021, parallel density gradient has little effects on the ECM processes. Additionally, observed frequency separation between adjacent zebra-structure stripes are not always equidistant as well as observed zebra-structure stripes can be present at noninteger harmonics, which cannot be explained by a simple cyclotron harmonic emission. Nonetheless, our results indicate that the Doppler effect due to the drifting motion of electron beams can influence the frequency separation between adjacent stripes, and lead to noninteger harmonic emissions in particular when their propagating angles  $\theta$  are small.

Note that more widespread and standard mechanism for the solar radio bursts with zebra-stripe pattern are based on the double plasma resonance (DPR) or Bernstein instabilities (Winglee & Dulk 1986; Benáček & Karlický 2018; Karlický et al. 2022). Actually, the DPR, Bernstein, and ECM instabilities have the same free energy, i.e., a positive velocity gradient in the electron distribution function perpendicular to the ambient magnetic field  $\partial f/\partial v_{\perp} > 0$ . The main difference among these three instabilities is that the DPR and Bernstein (ECM) instability is applied to weakly (strongly) magnetized plasmas with  $\omega_{ce} < \omega_{pe}$  ( $\omega_{ce} > \omega_{pe}$ ) and trigger excitation of the electrostatic upper-hybrid and Bernstein waves (electromagnetic X- and O-mode waves), respectively. How the electrostatic upper-hybrid and Bernstein waves transform into electromagnetic waves and are observed as radio zebra-structure emissions should be considered for the DPR and Bernstein mechanism to explain the formation of the solar radio bursts with zebra-stripe pattern (Li et al. 2021). The ECM instability can, however, excite electromagnetic waves directly. The DPR and Bernstein model are more accredited for the emission mechanism of the solar radio bursts with zebra-stripe pattern since these models fulfill well the standard plasma model of the solar corona with  $\omega_{ce} < \omega_{pe}$  (Wild 1985). However it is still possible that  $\omega_{ce} > \omega_{pe}$  exists in density cavities of the solar corona due to ubiquitous Alfvénic turbulence (Wu et al. 2014; Chen et al. 2017) as well as in the low-density separatrices of magnetic reconnection (Drake et al. 2003; Pritchett & Coroniti 2004; Muñoz & Büchner 2018). Observations by Régnier 2015; Morosan et al. 2016 have provided the existence of  $\omega_{ce} > \omega_{pe}$  in the solar corona above the core of active regions. All above posts the ECM mechanism (in addition to the DPR and Bernstein models) as one of the valid candidates for the interpretation of the solar radio bursts with zebra-stripe pattern.

Additionally, note that there still needs further investigations to discover whether the nonlinear wave-wave interactions occur under the presence of the forward and backward Langmuir waves in these inhomogeneous plasmas. But what we ascertain is

that the nonlinear wave-wave interactions of the forward and backward Langmuir waves do not contribute much to these most strongly excited electromagnetic waves, which are produced, for sure, by the ECM emission processes in this study. In spite of the wave conversion efficiency of the plasma emission process, scattering of forward Langmuir waves to the backward Langmuir waves more likely occurs in a cold and dense plasma. Most space plasmas, however, stay hot and tenuous in the solar corona. The ECM emission processes are, hence, expected more for the generation of radio bursts from the solar corona, where frequency requirement for a strong ECM emission, i.e.,  $\omega_{ce} > \omega_{pe}$ , could be easily satisfied in the presence of Alfvén waves (Wu et al. 2014).

The present research at PMO was supported by the Strategic Priority Research Program of the Chinese Academy of Sciences under grant No.XDB0560000, the project of National Natural Science Foundation of China No.12003073, 42174195, 11873018, and 11790302. And Jan Benáček acknowledges the support by the German Science Foundation (DFG) project BE 7886/2-1. We gratefully acknowledge the developers of the ACRONYM code, the Verein zur Förderung kinetischer Plasmasimulationen e.V., as well as the computing resources in the Max Planck Computing and Data Facility (MPCDF) at Garching, Germany, and the Max Planck Institute for Solar System Research, Germany.

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